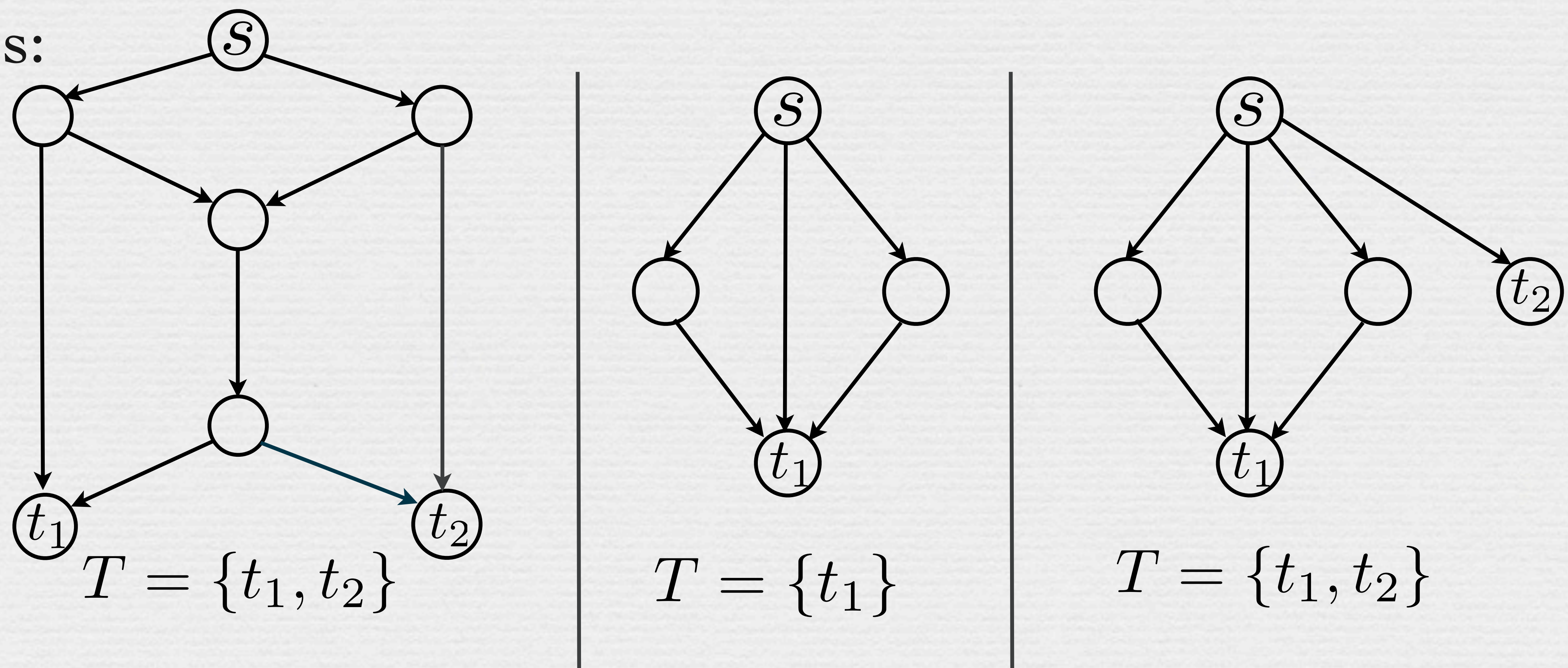
The Multicast Problem: definition

Definition

A multicast problem is a triple (G,s,T) such that:

- $G=(V,E)$ is an acyclic directed graph;
- $s \in V$ is a vertex, called the *source;*
- $T = \{t_1, \ldots, t_k\} \subseteq V$ is a set of *target* vertices.

Examples:



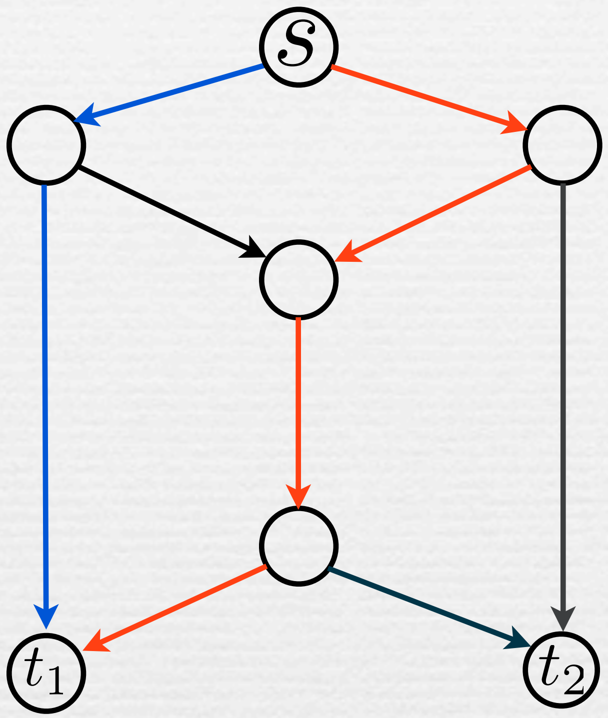
The Main Theorem of Network Coding

Theorem:

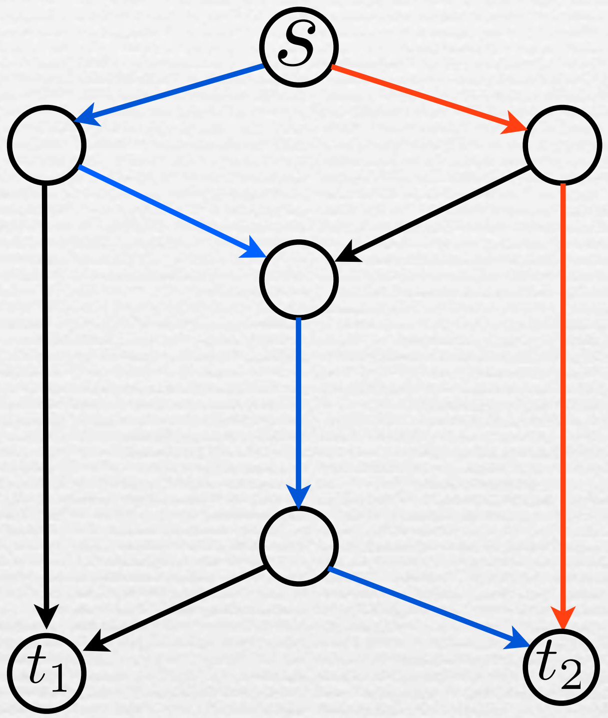
Let r be an integer. A multicast problem (G,s,T) is solvable at rate r using coding if and only if the following condition holds:

($\star$) for each vertex $t \in T$, there exist r edge-disjoint paths from s to t.

Example 1:

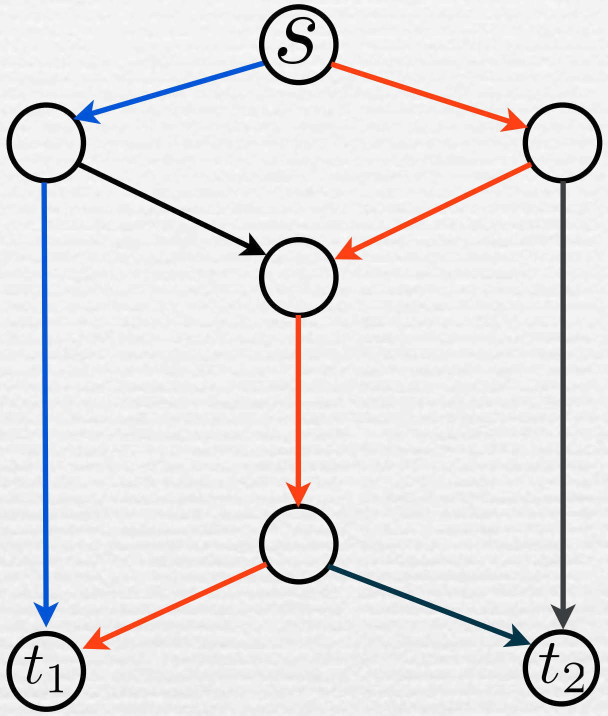


2 edge-disjoint paths from s to t_1

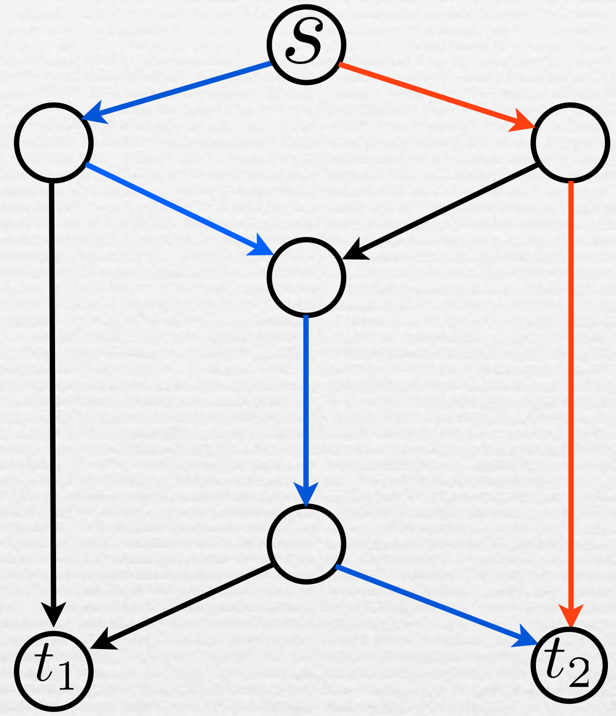


2 edge-disjoint paths from s to t_2

Example 1: condition (★) holds for $r = 2$



2 edge-disjoint paths
from s to t_1



2 edge-disjoint paths
from s to t_2

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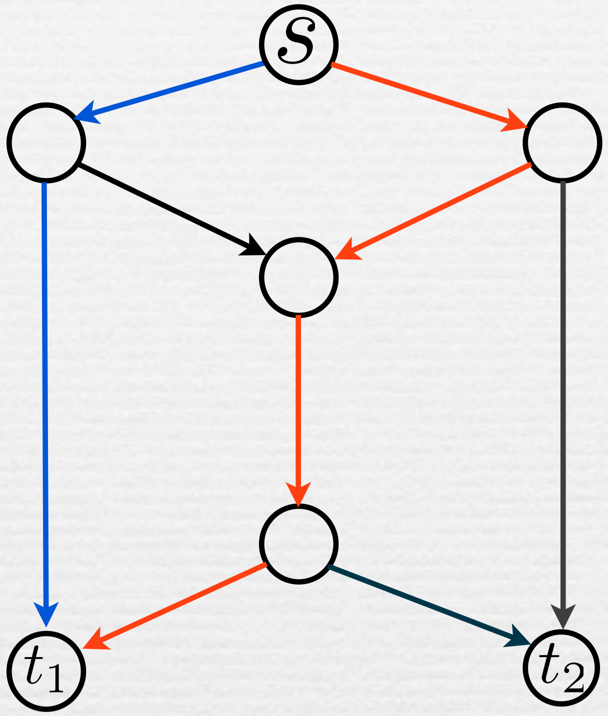
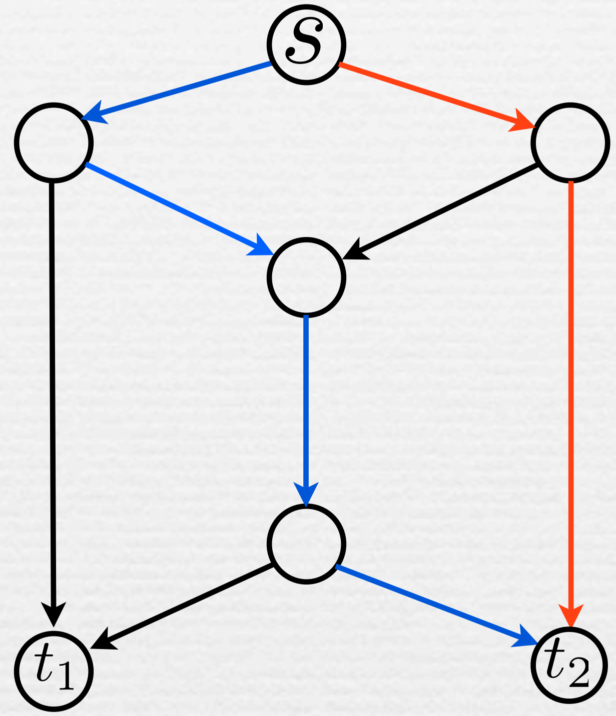
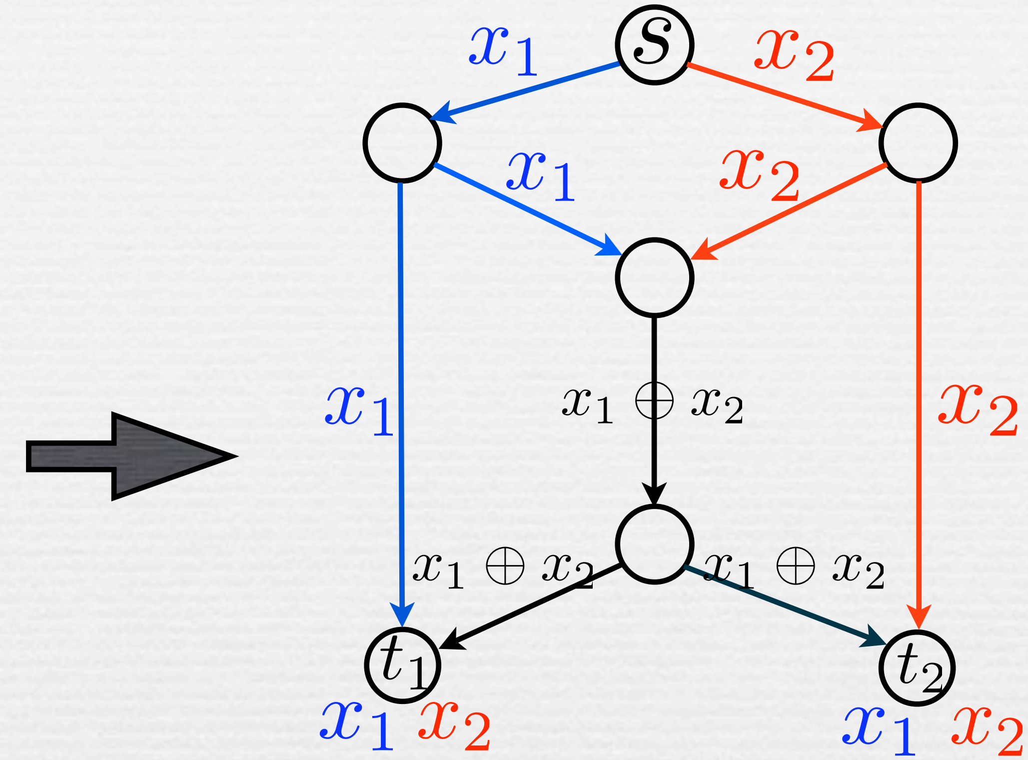


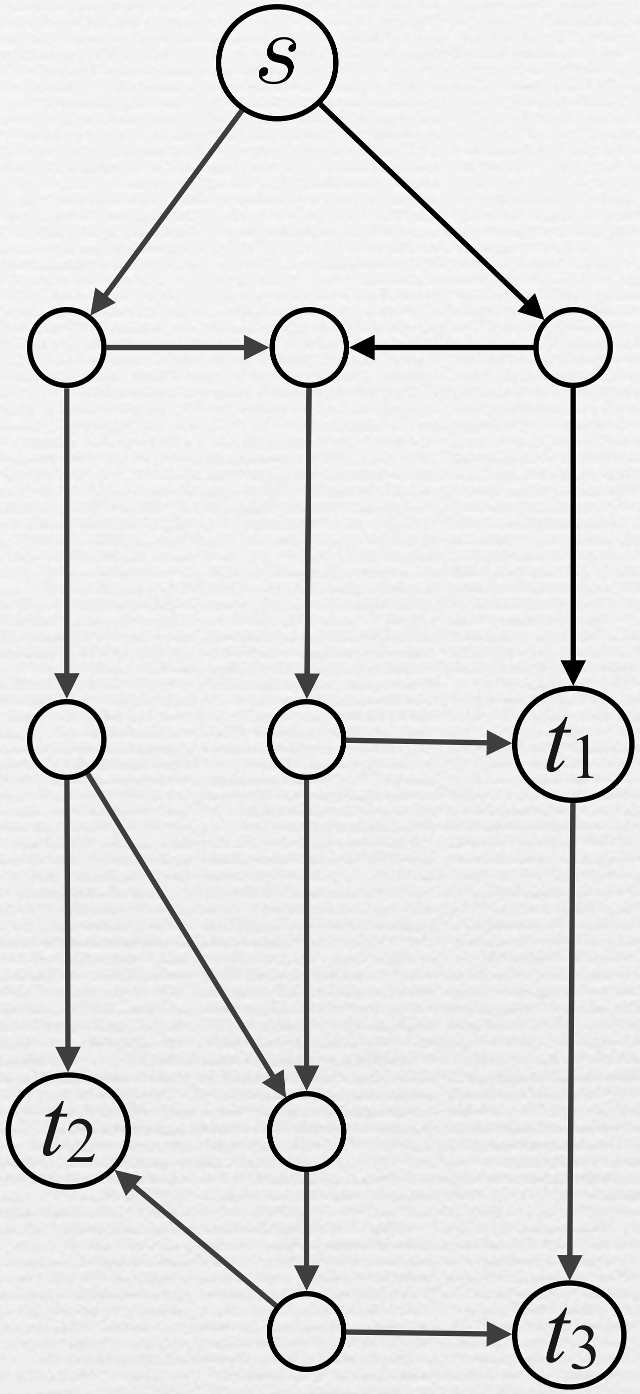
2 edge-disjoint paths from s to t_1



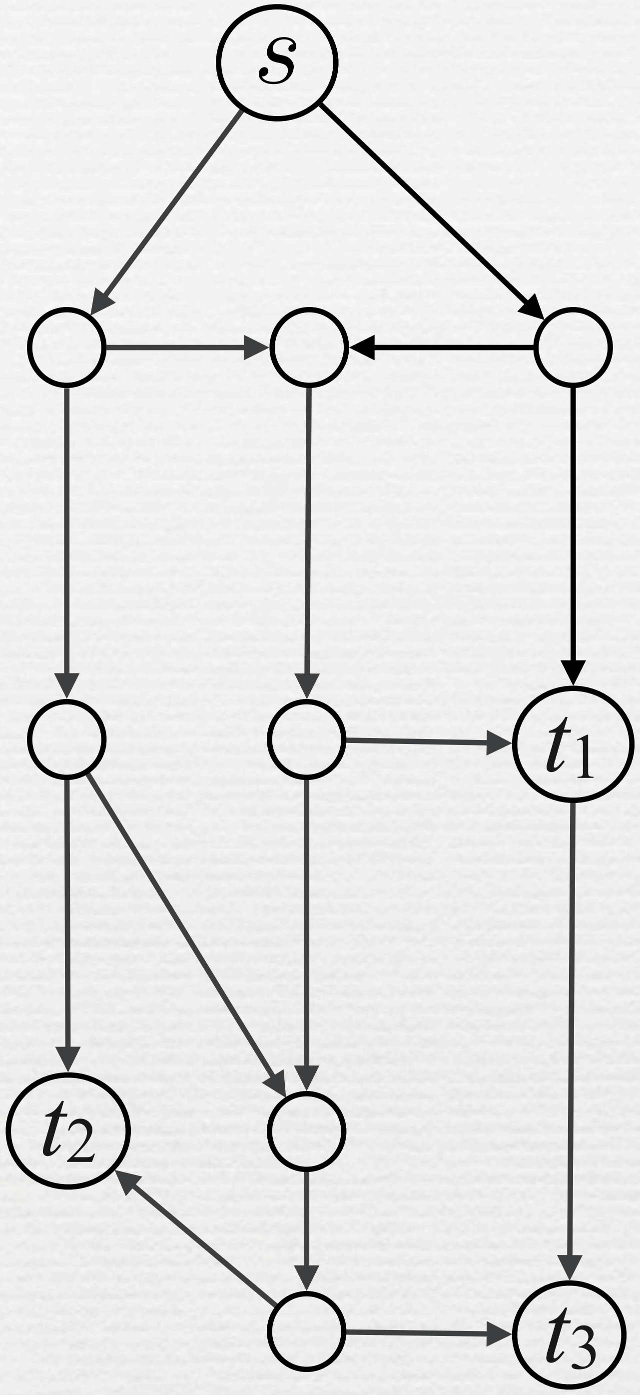
2 edge-disjoint paths from s to t_2



solvable at rate 2
(with the alphabet GF(2))



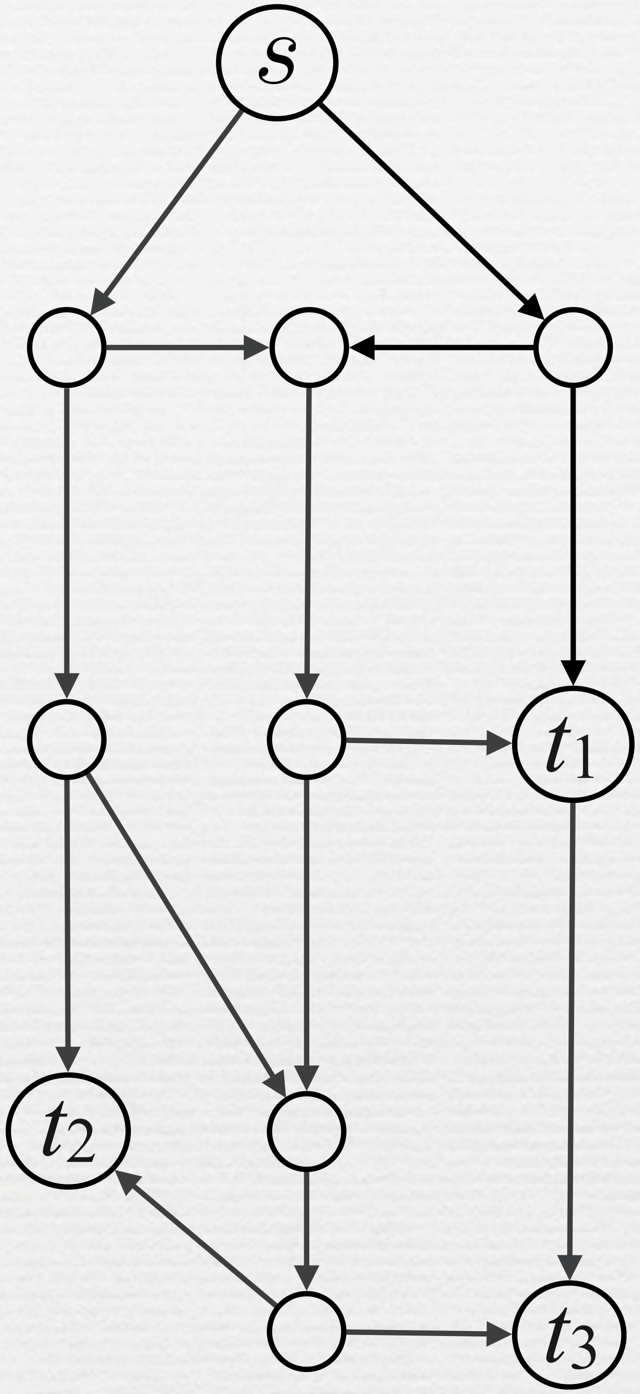
Example 2



Example 2

condition (★) holds for r = ?

(★) for each vertex $t \in T$, there exist r edge-disjoint paths from s to t.



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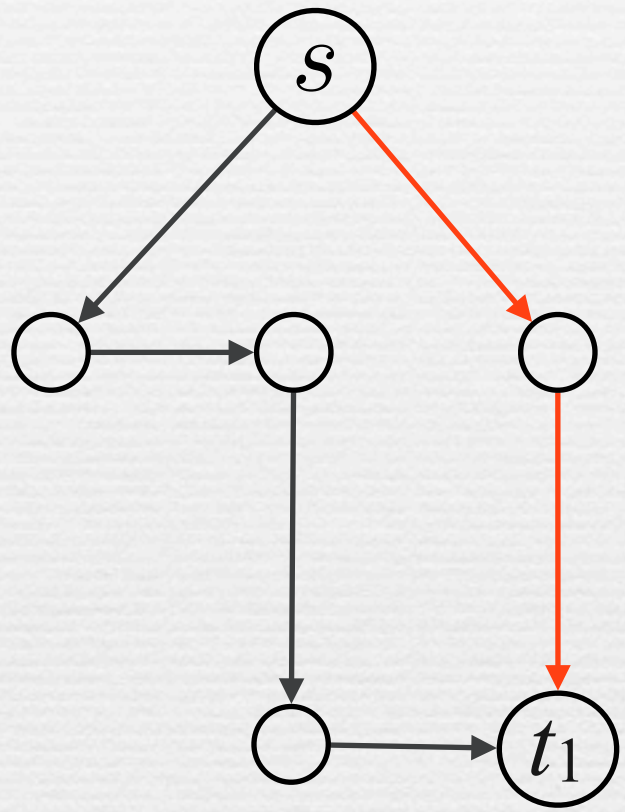

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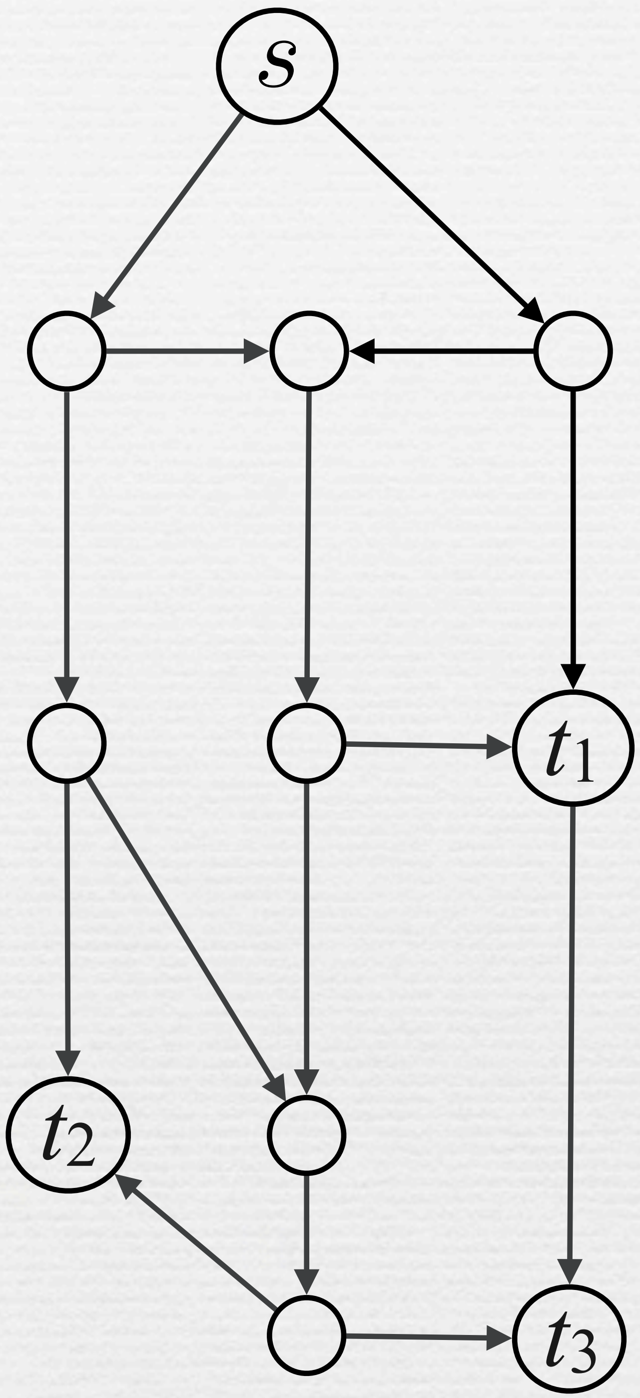

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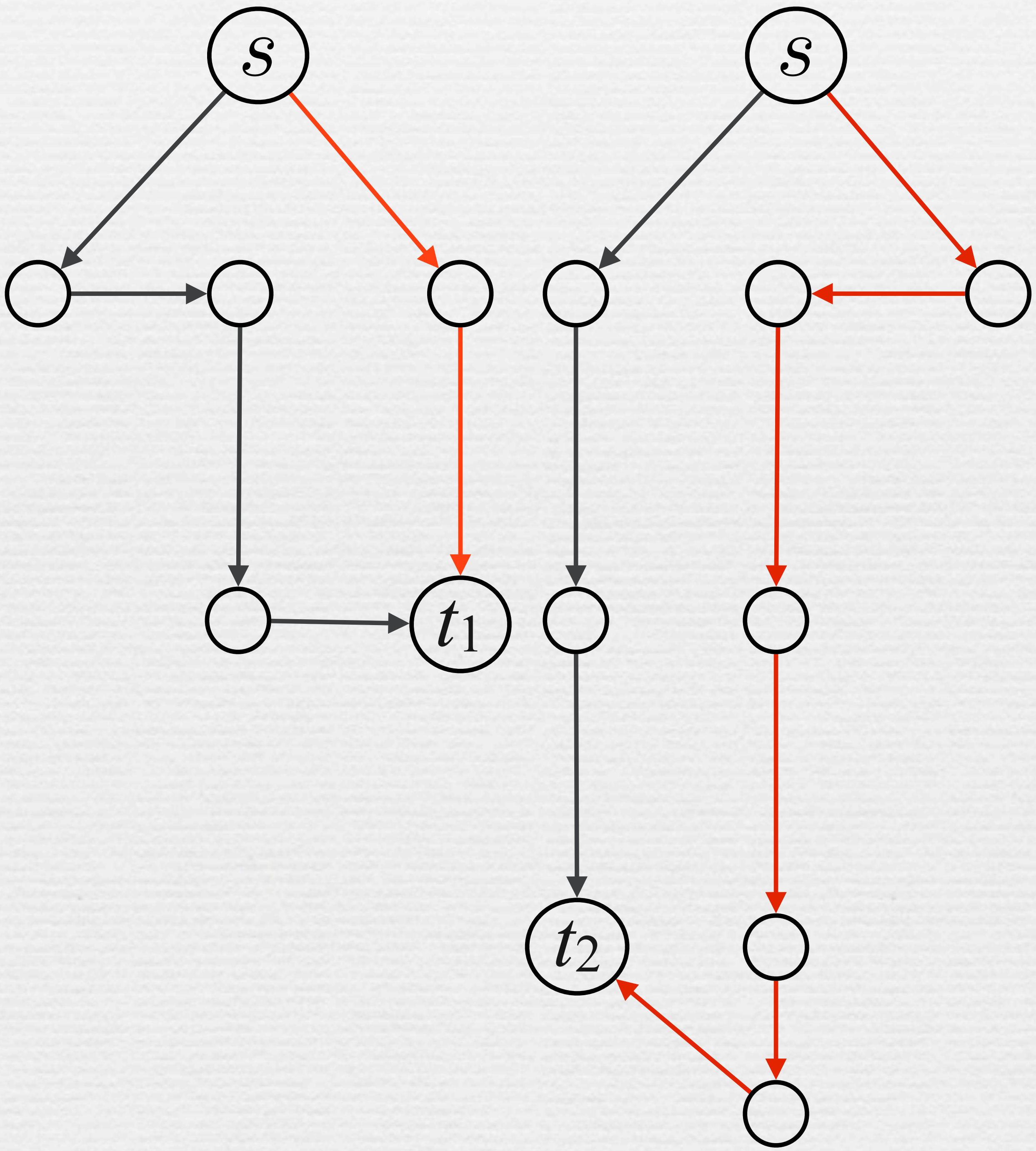

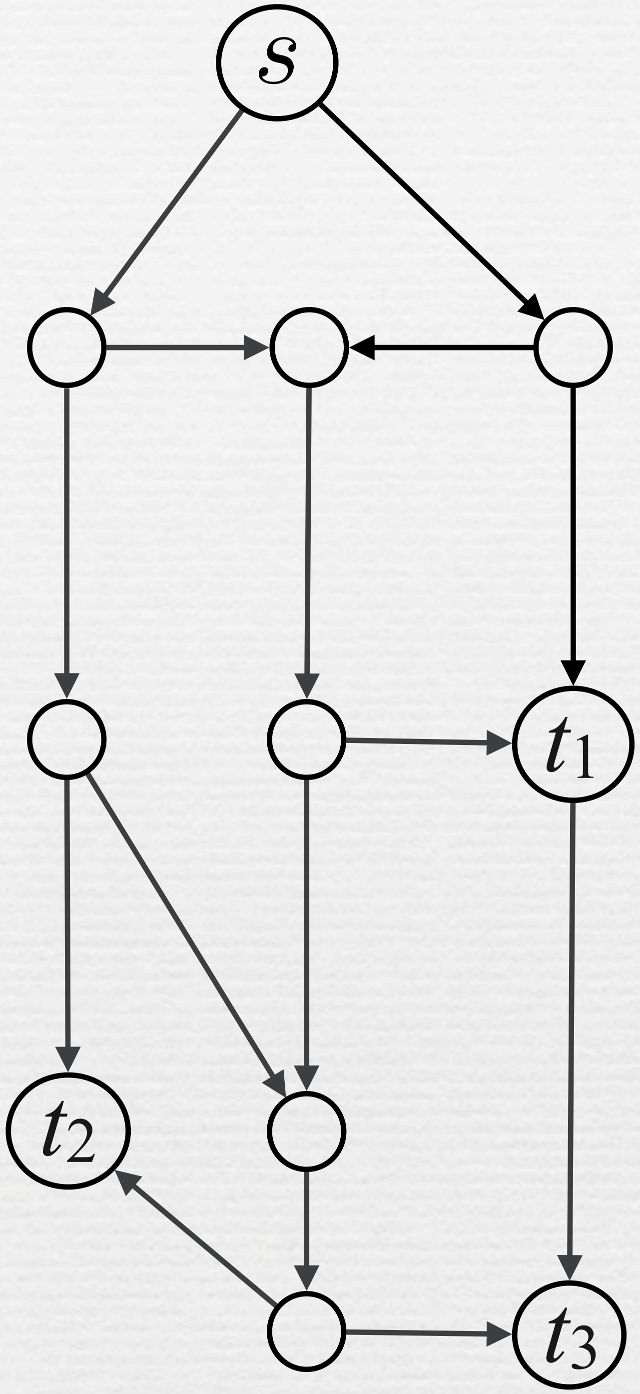

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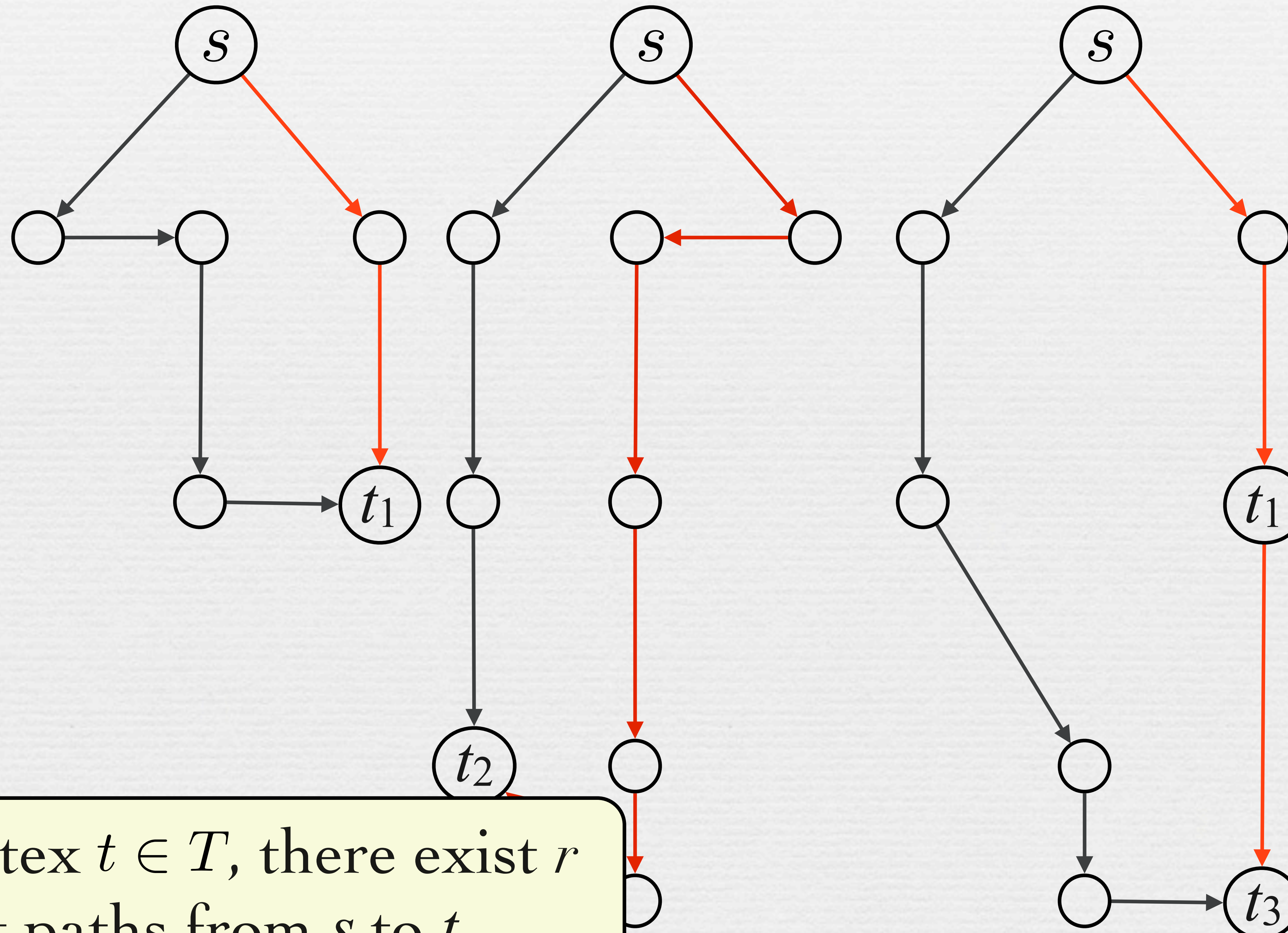
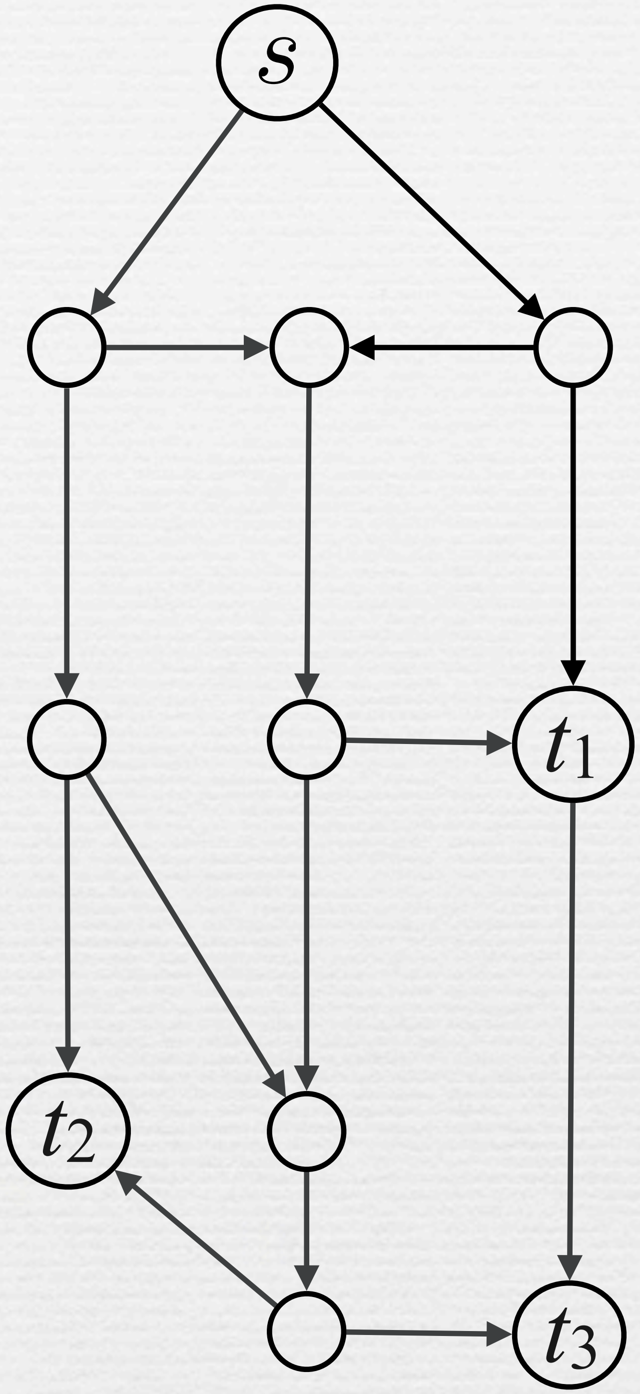


(★) for each vertex $t \in T$, there exist r edge-disjoint paths from s to t.



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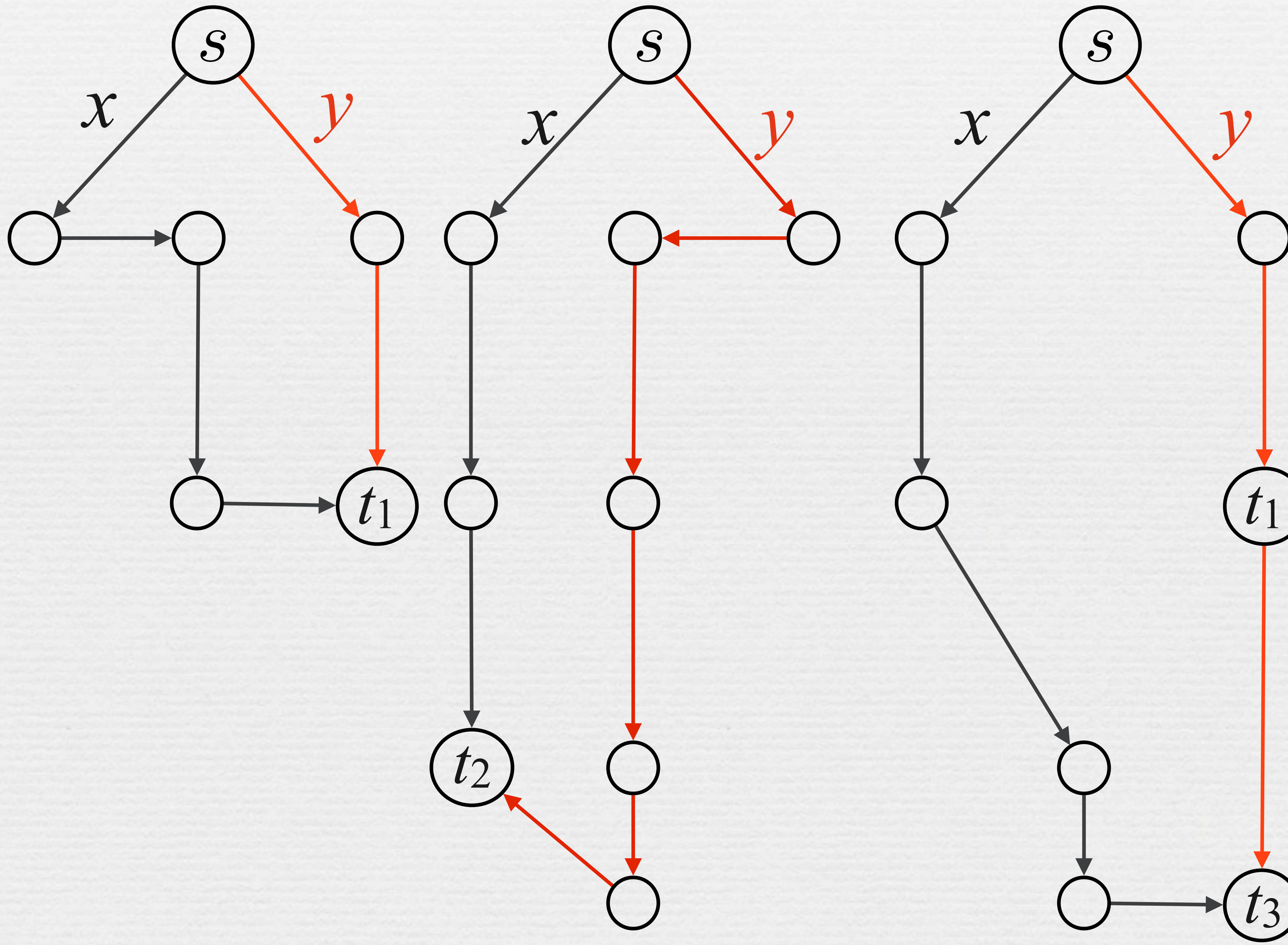

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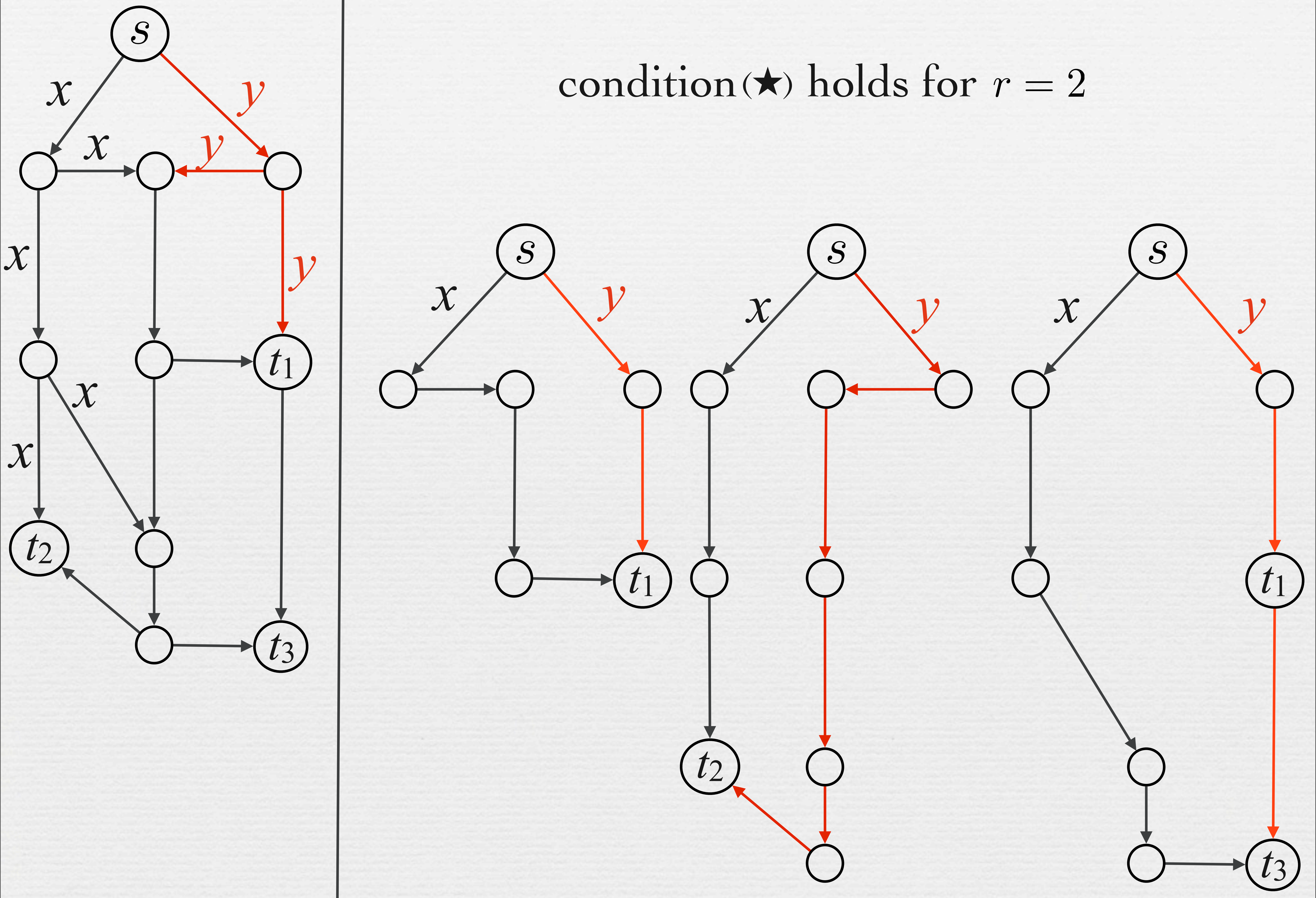
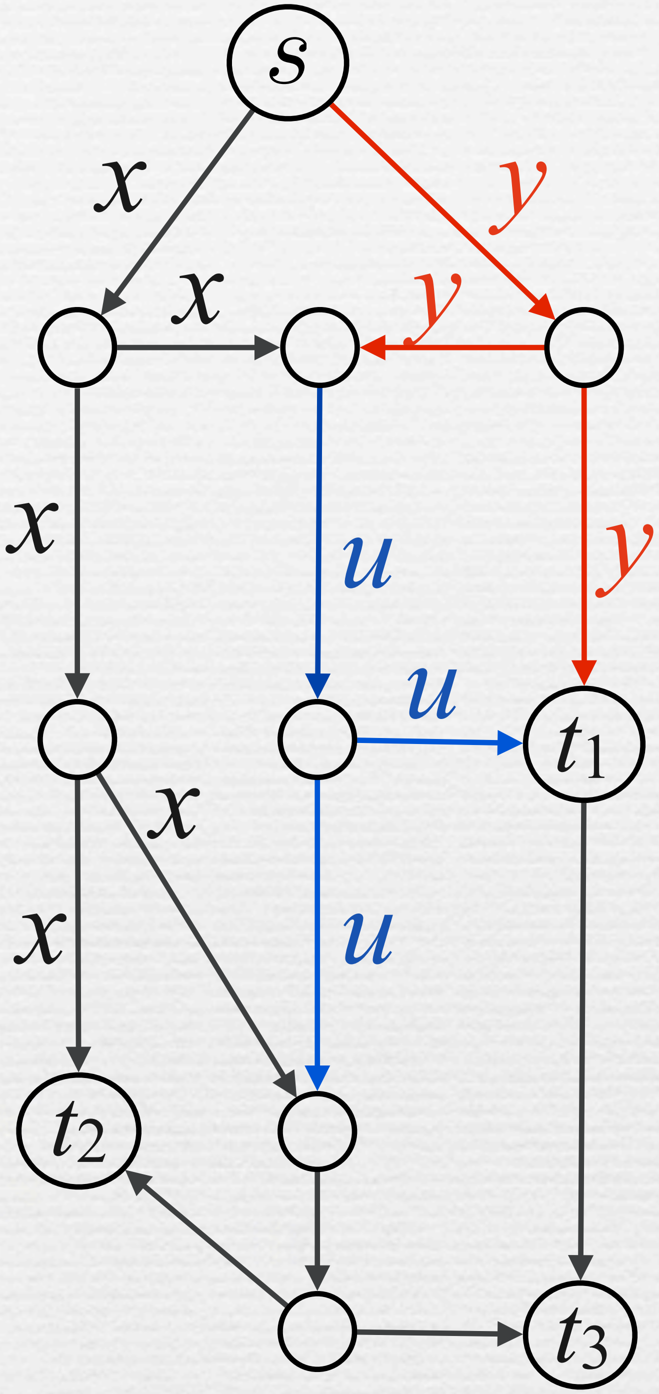
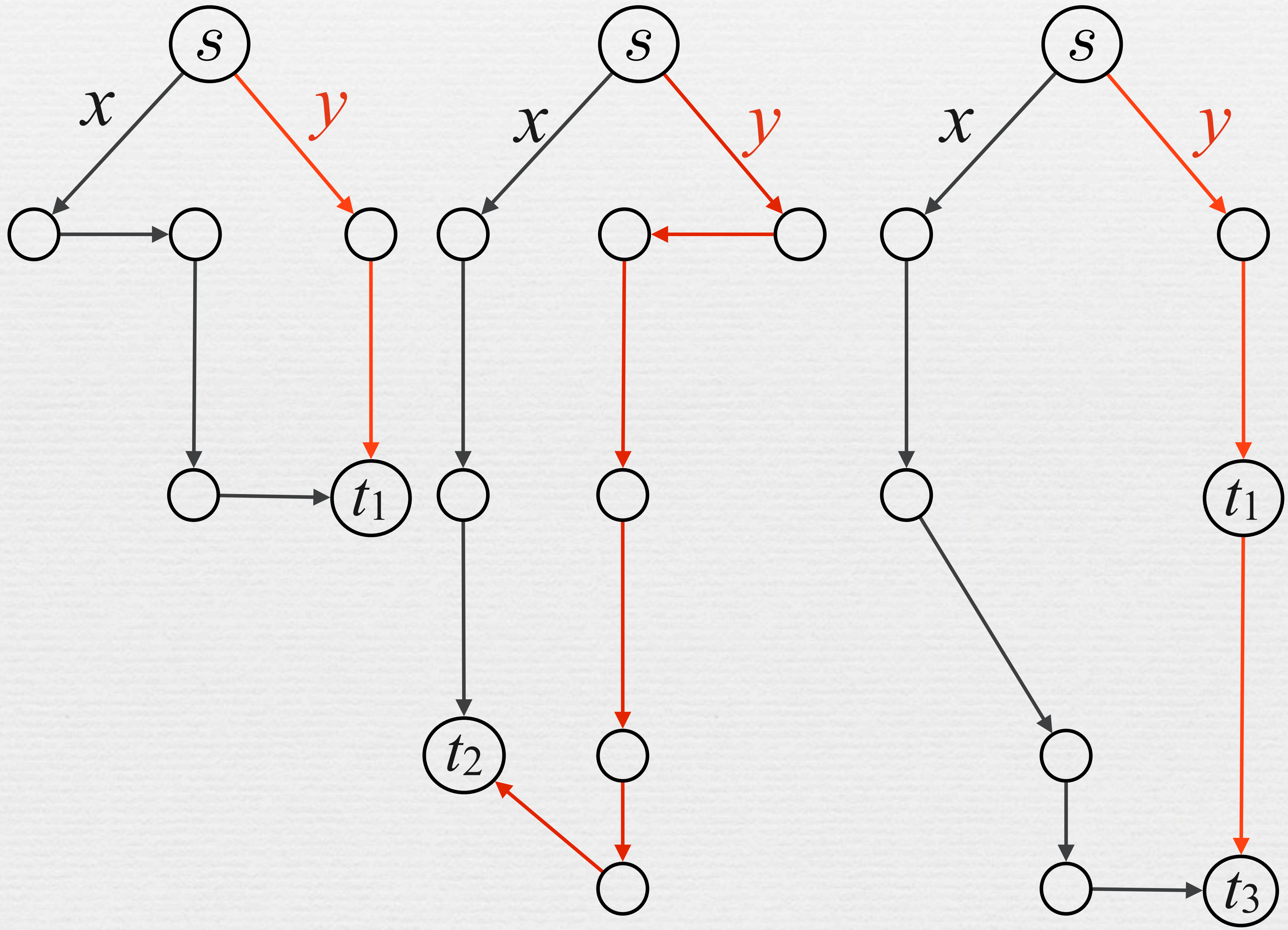
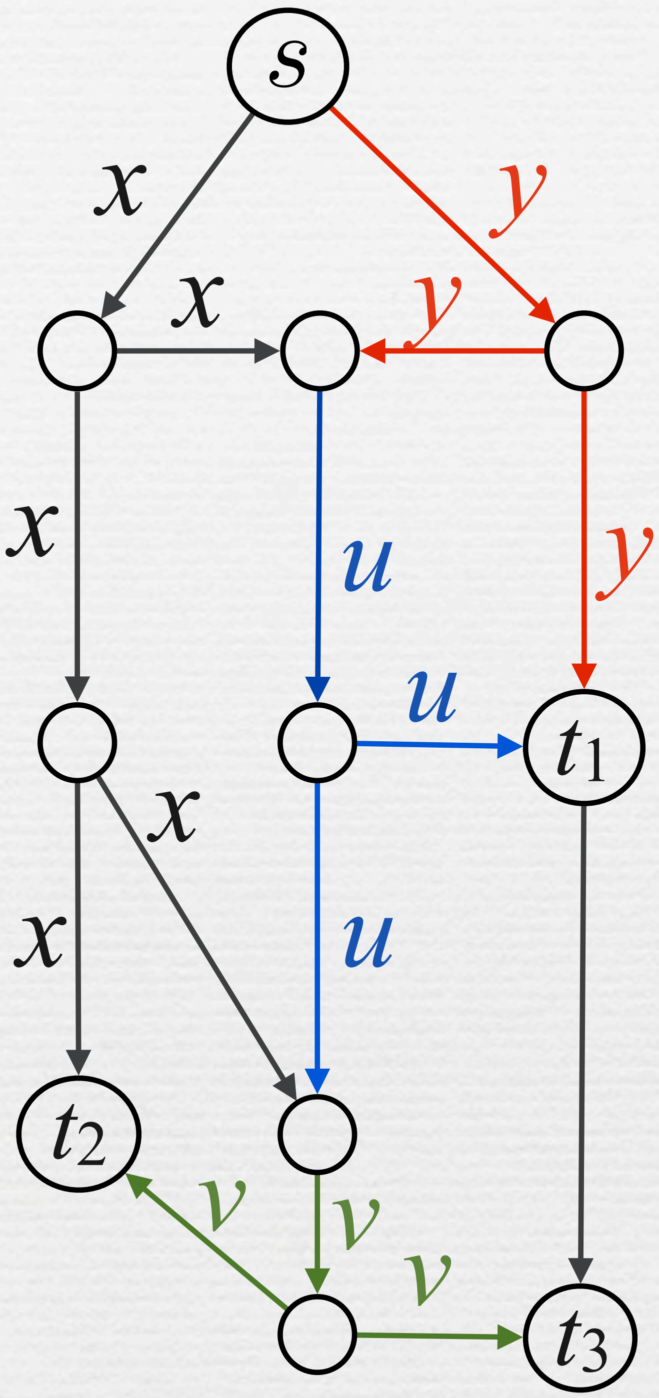




$u = ax + by$

condition (★) holds for $r = 2$

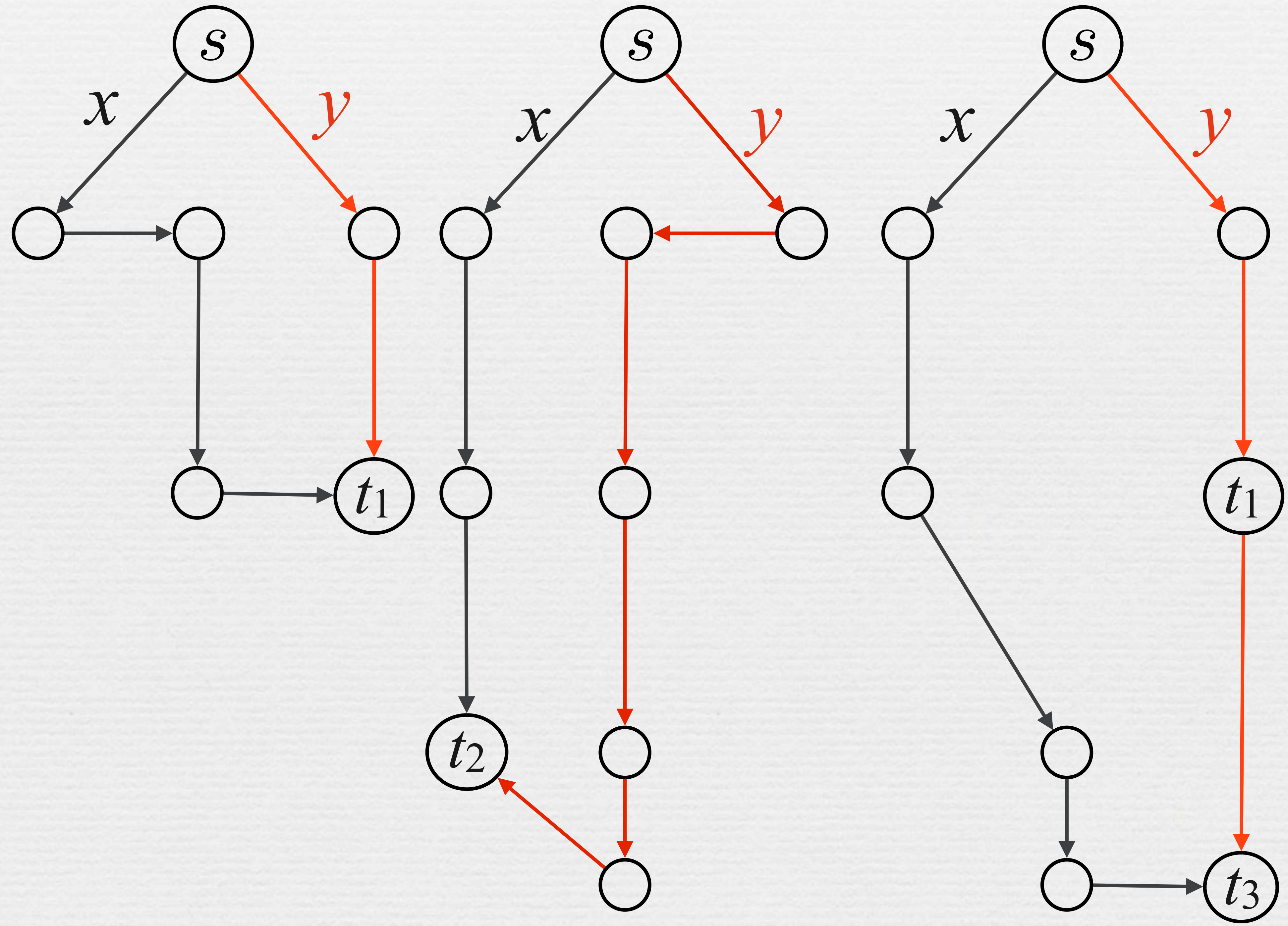


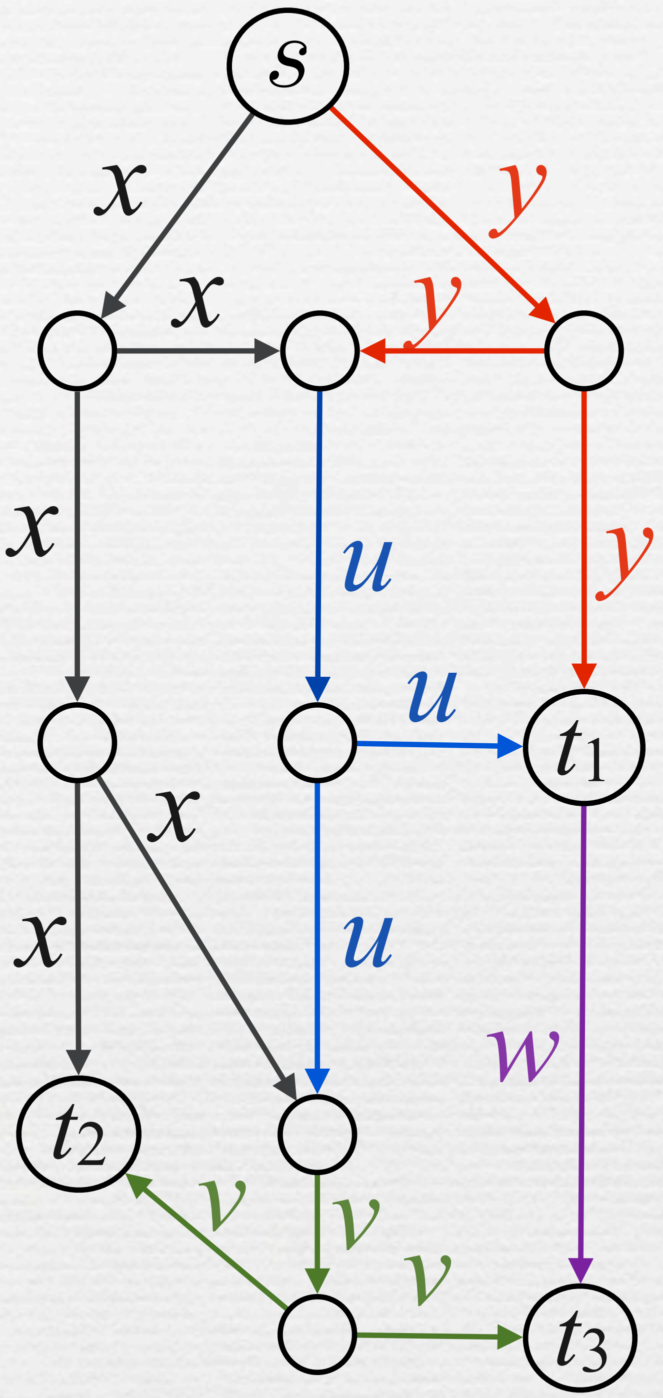


$u = ax + by$

$v = cx + du$

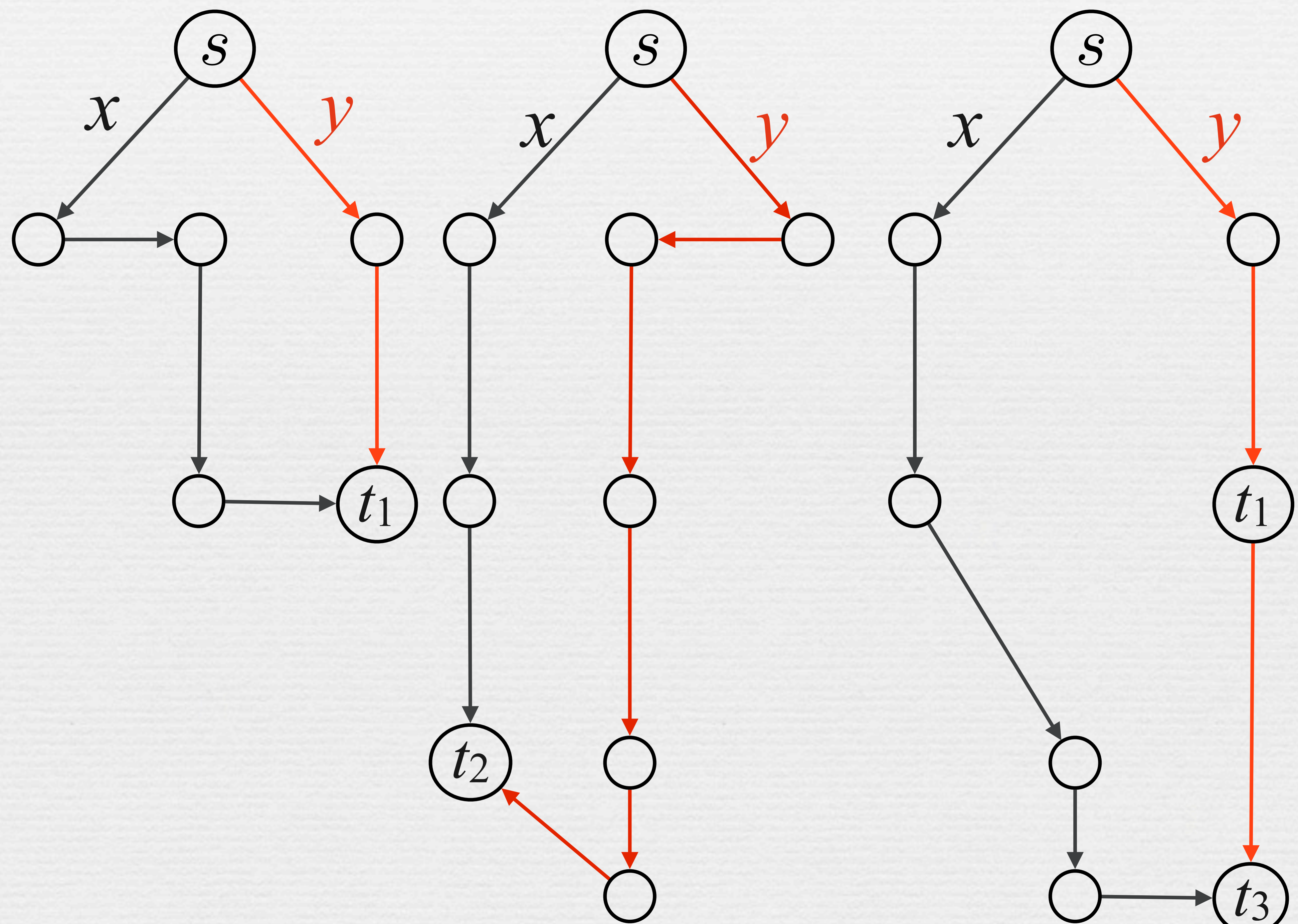
condition (★) holds for $r = 2$

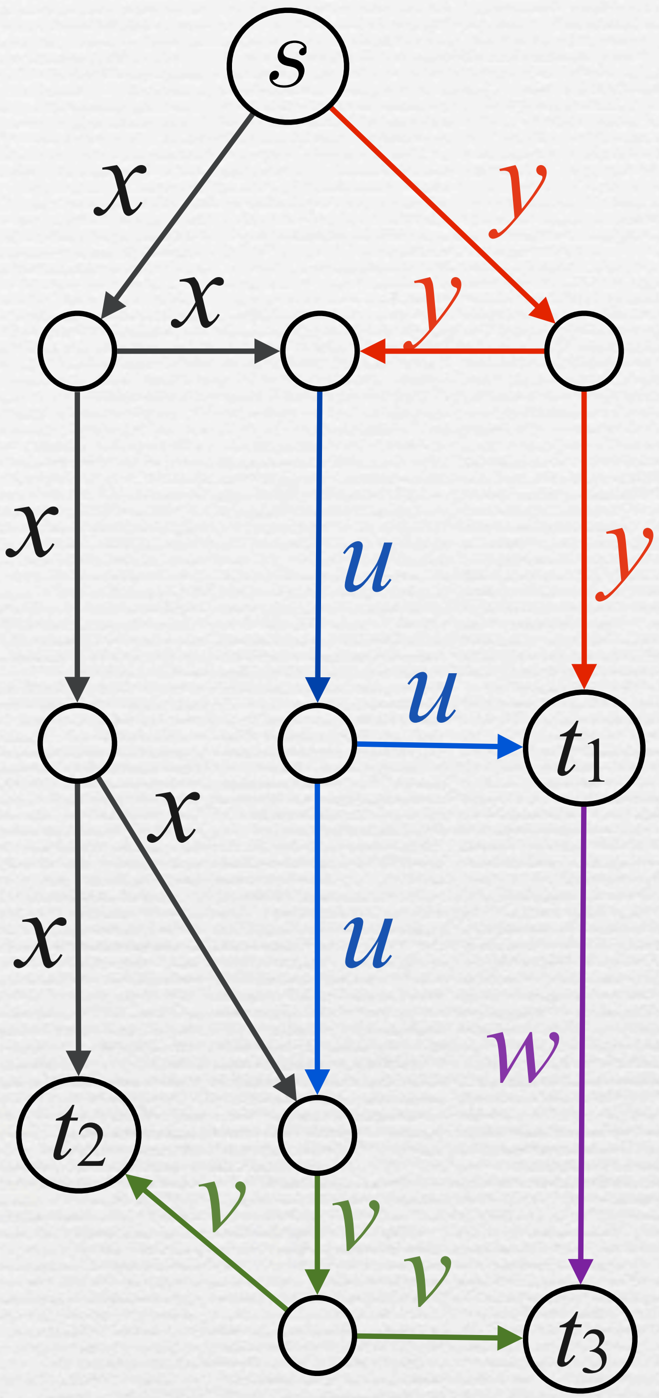




$$u = ax + by$$
$$v = cx + du$$
$$w = eu + fy$$

condition (★) holds for $r = 2$





$u = ax + by$

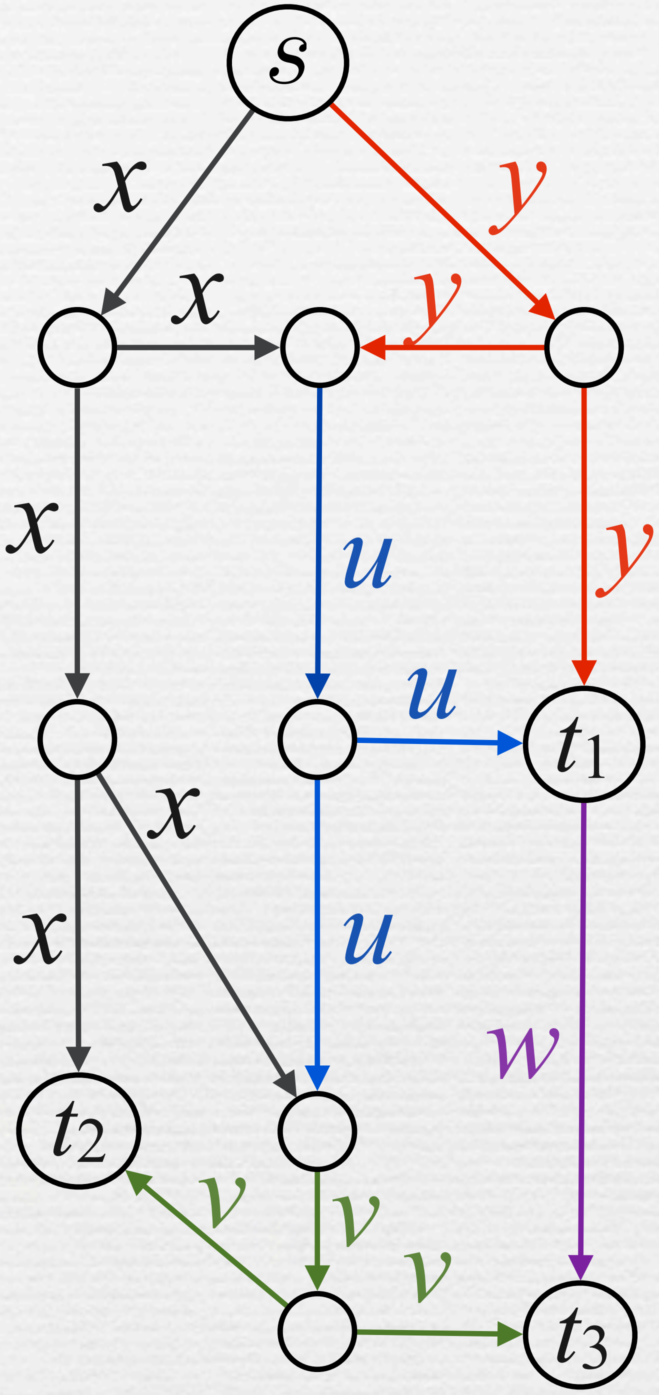
$v = cx + du$

$w = eu + fy$

at t_1: $$\begin{pmatrix} u \\ y \end{pmatrix} = \overbrace{\begin{pmatrix} a & b \\ 0 & 1 \end{pmatrix}}^{A_1} \begin{pmatrix} x \\ y \end{pmatrix}$$

at t_2: $$\begin{pmatrix} x \\ v \end{pmatrix} = \overbrace{\begin{pmatrix} 1 & 0 \\ c + ad & bd \end{pmatrix}}^{A_2} \begin{pmatrix} x \\ y \end{pmatrix}$$

at t_3: $$\begin{pmatrix} v \\ w \end{pmatrix} = \overbrace{\begin{pmatrix} c + ad & bd \\ ae & be + f \end{pmatrix}}^{A_3} \begin{pmatrix} x \\ y \end{pmatrix}$$



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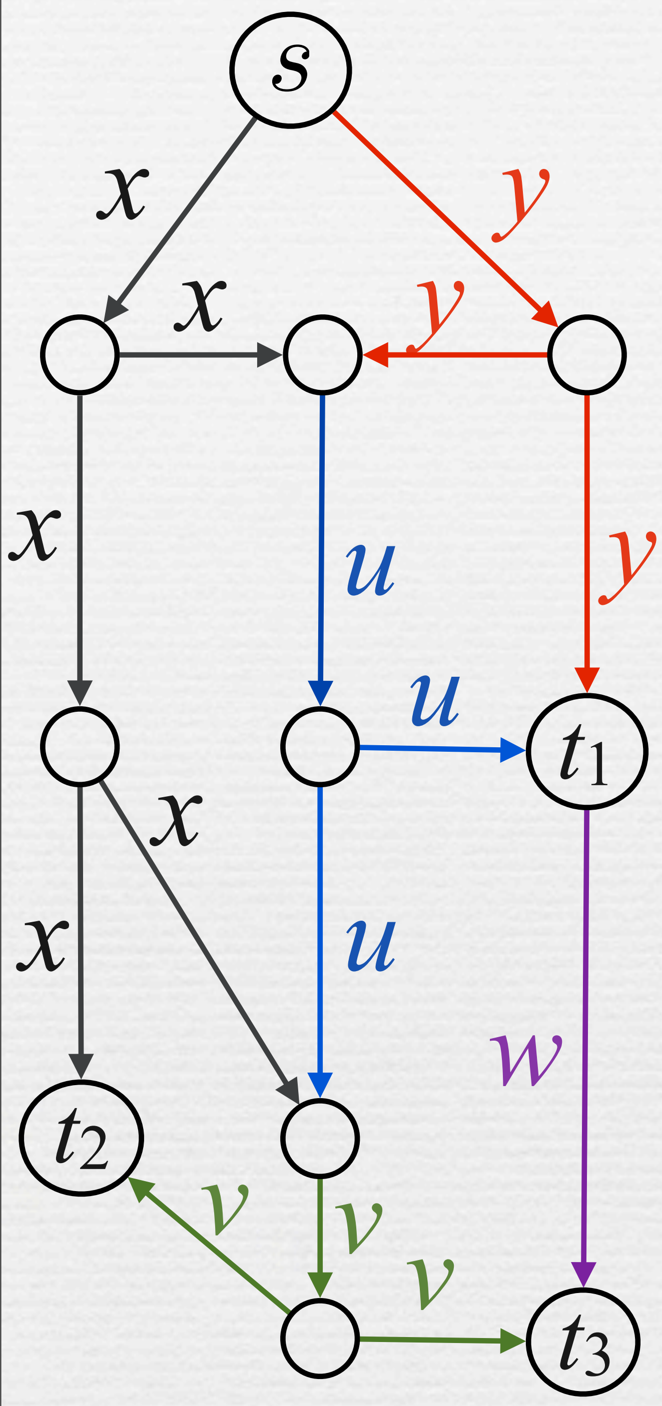
choose a, b, c, d, e and f such that the three matrices are invertible



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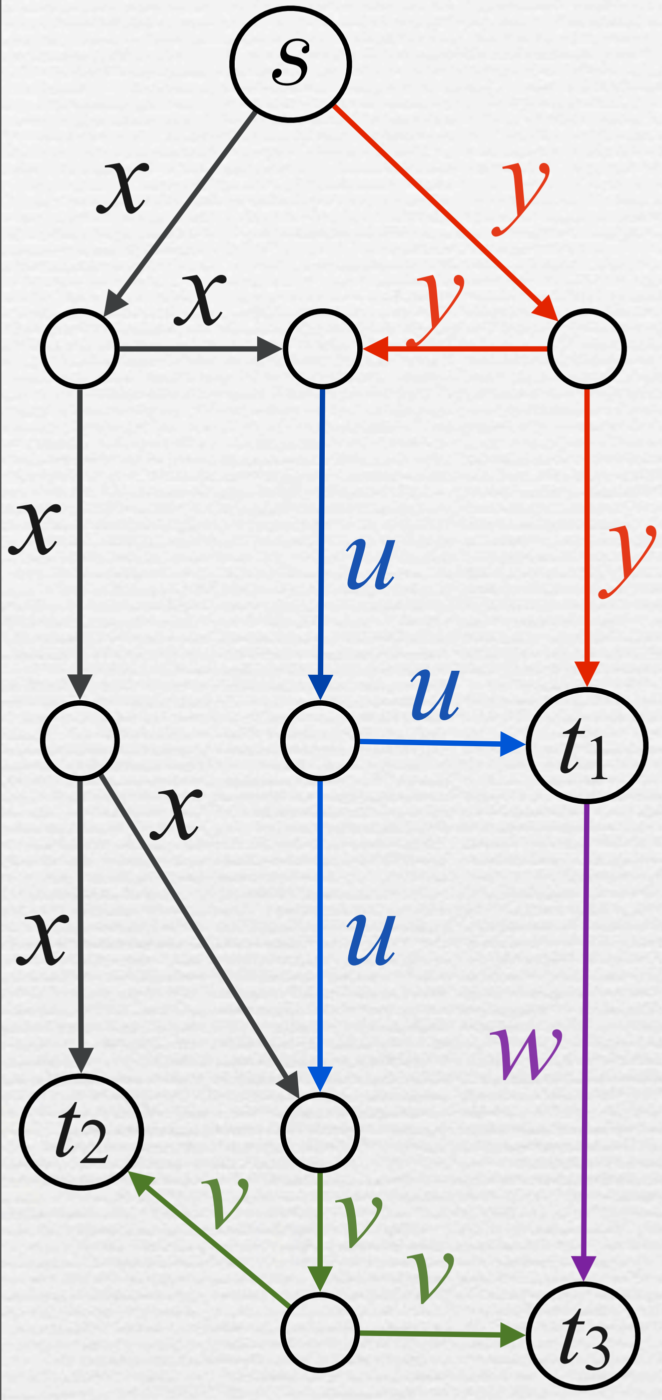
example: $a=b=d=f=1$ and $c=e=0$



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(this gives a solution over any alphabet GF(q))