Algorithmic Aspects of Communication
Lecture 1

Network Coding

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A Motivating Example

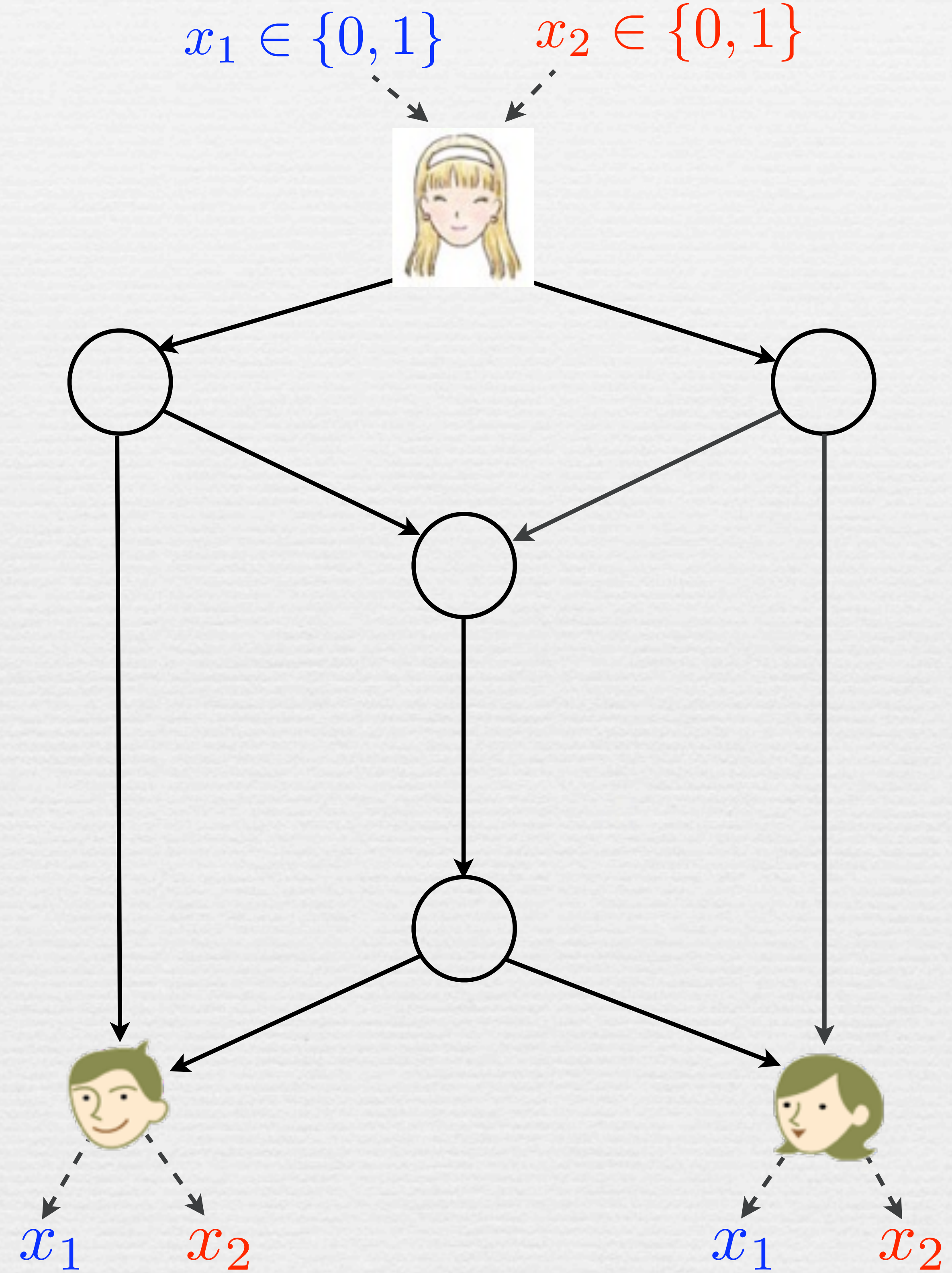
- communication network
- one sender: Alice 
- two receivers: Bob  and Chris 

input: Alice receives two bits x_1 and x_2

Task

send x_1 and x_2 to both Bob and Chris

- each edge has capacity 1 bit and can be used <u>only once</u>



A Motivating Example: attempt using routing

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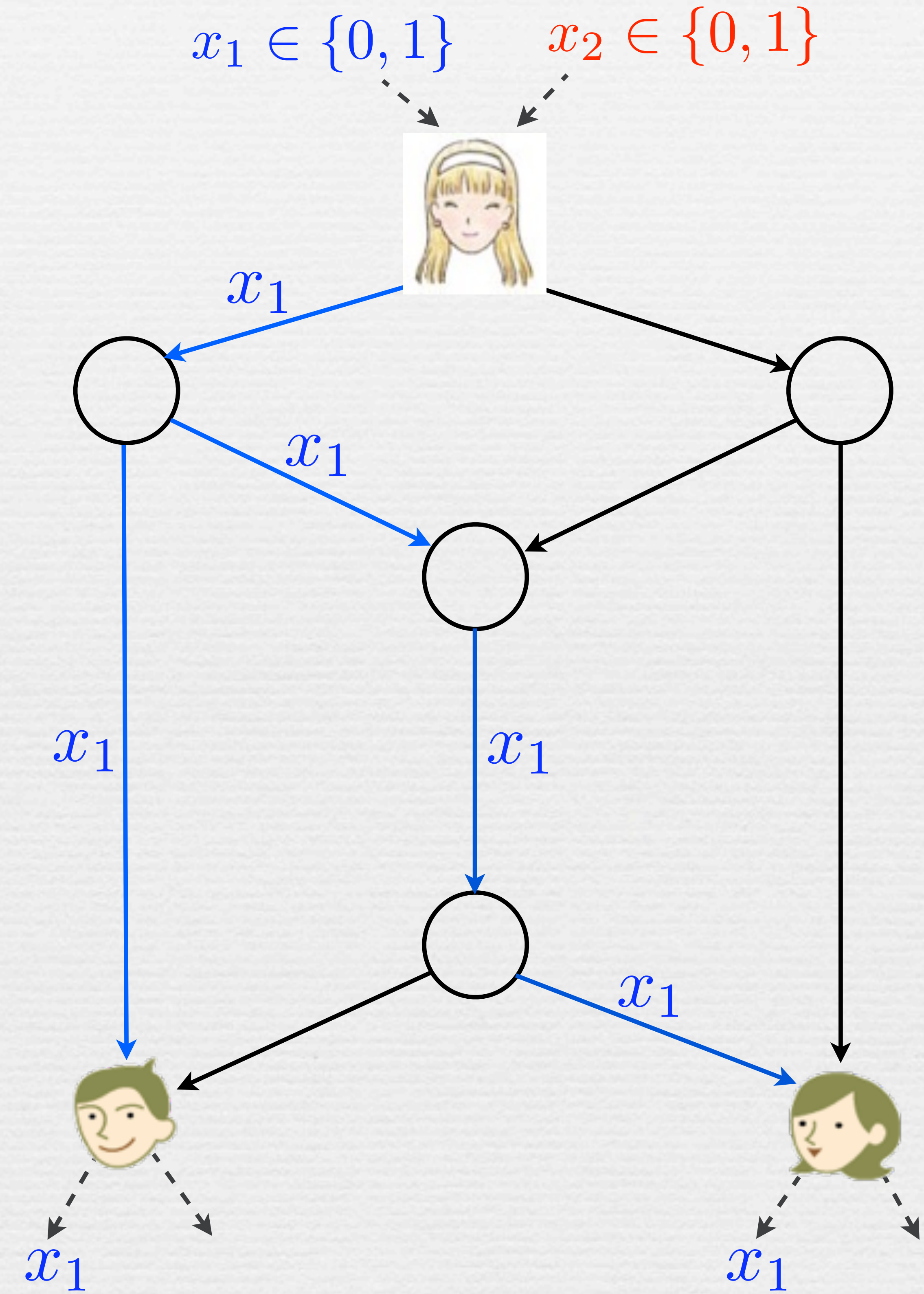

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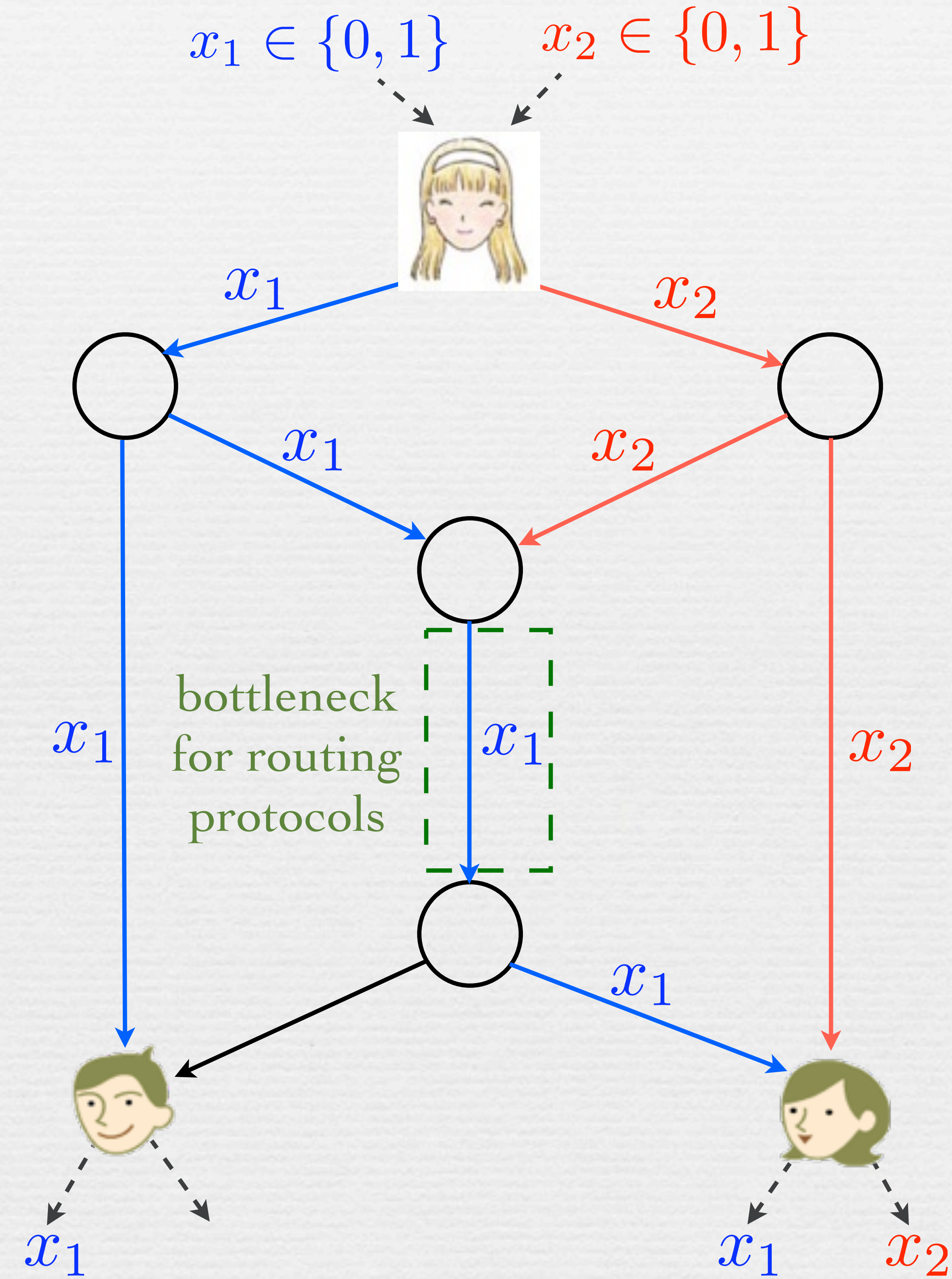


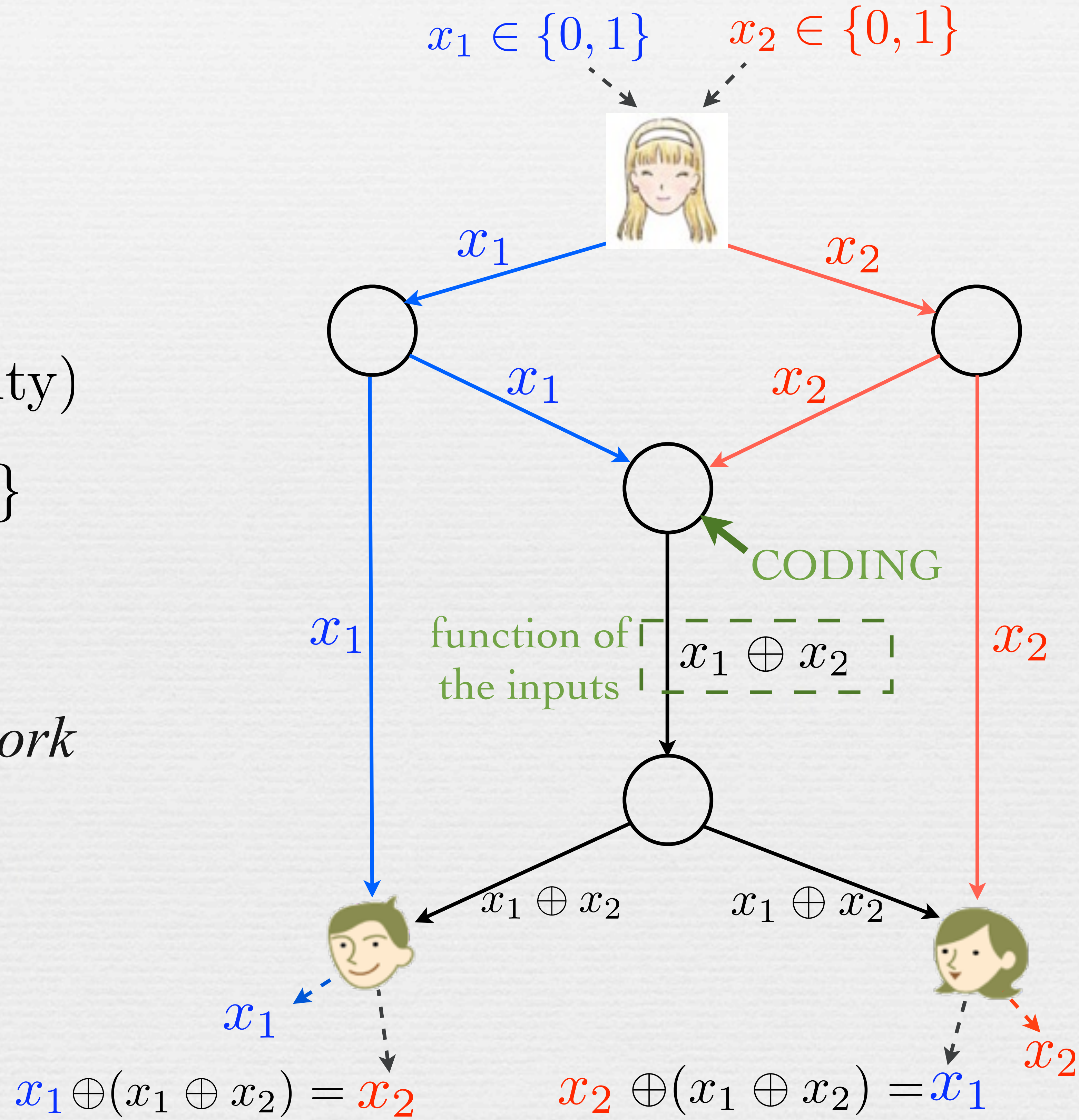
A Motivating Example: solution using coding

$\oplus$: exclusive-OR (bit parity)

$x \oplus x = 0$ for all $x \in \{0, 1\}$

our target: study of *network coding protocols*

relatively new topic (seminal paper in 2000)

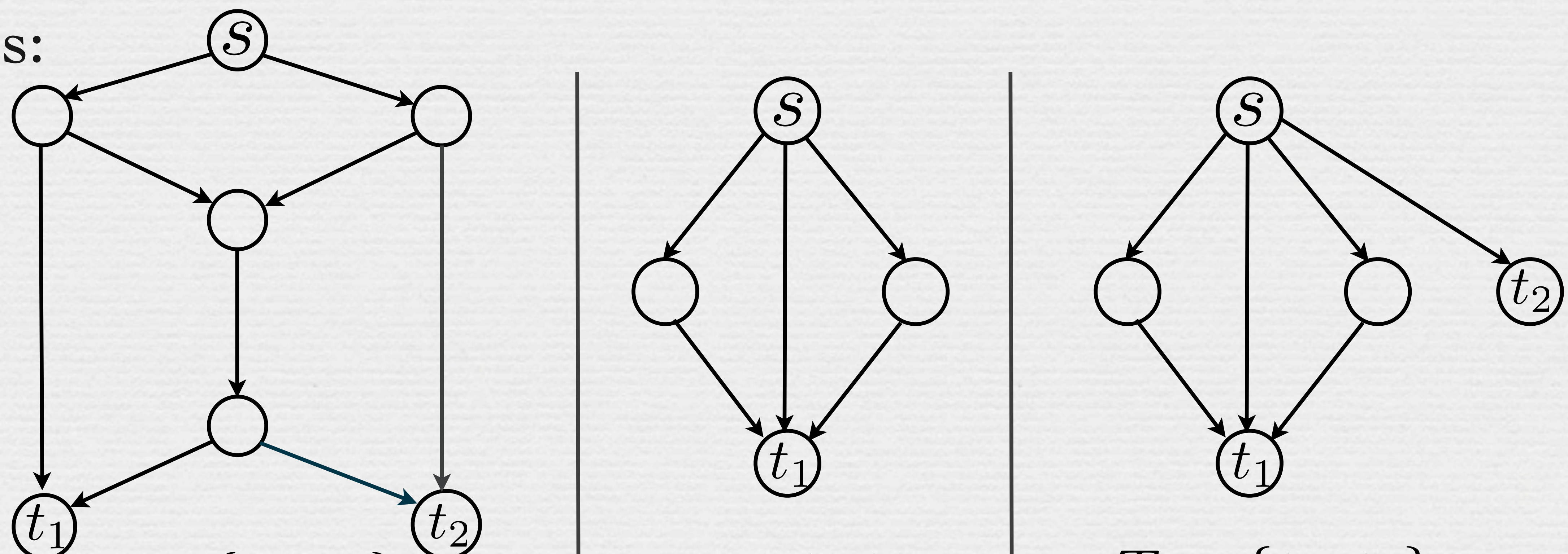


The Multicast Problem: definition

<u>Definition</u>

> A multicast problem is a triple (G,s,T) such that:
>
> - $G=(V,E)$ is an acyclic directed graph;
> - $s \in V$ is a vertex, called the *source;*
> - $T = \{t_1, \ldots, t_k\} \subseteq V$ is a set of *target* vertices.

Examples:



$T = \{t_1, t_2\}$ | $T = \{t_1\}$ | $T = \{t_1, t_2\}$

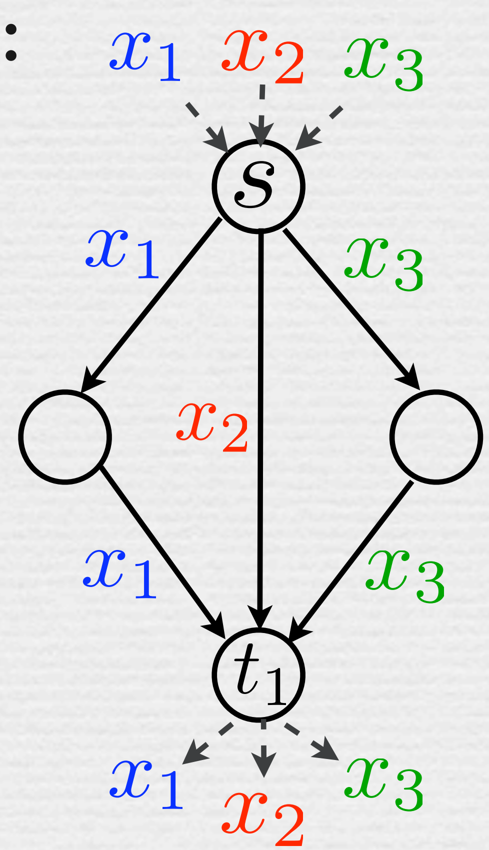
The Multicast Problem: solvability

- Each edge is supposed to have *capacity one bit.*

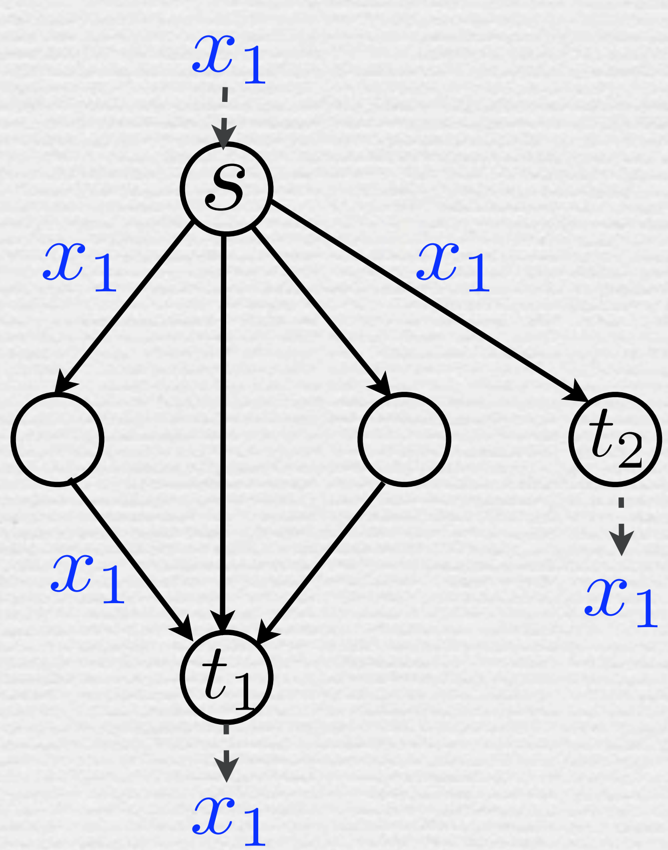
Definition

> Let r be an integer.
> We say that a multicast problem is *solvable at rate r* if r bits can be sent from s to all target vertices $t \in T$.

Examples:



solvable at rate 3



solvable at rate 1 (but not at rate 2)

The Main Theorem of Network Coding

Theorem:

Let r be an integer. A multicast problem (G,s,T) is solvable at rate r using coding if and only if the following condition holds:

($\star$) for each vertex $t \in T$, there exist r edge-disjoint paths from s to t.

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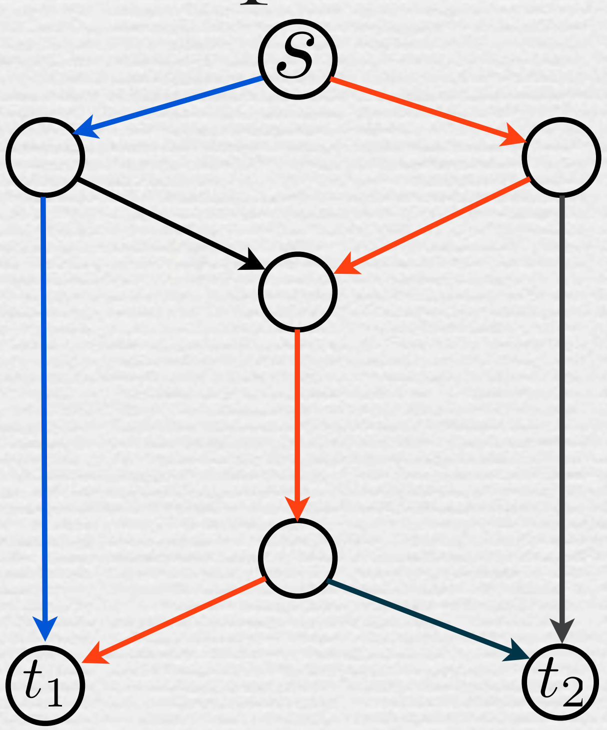
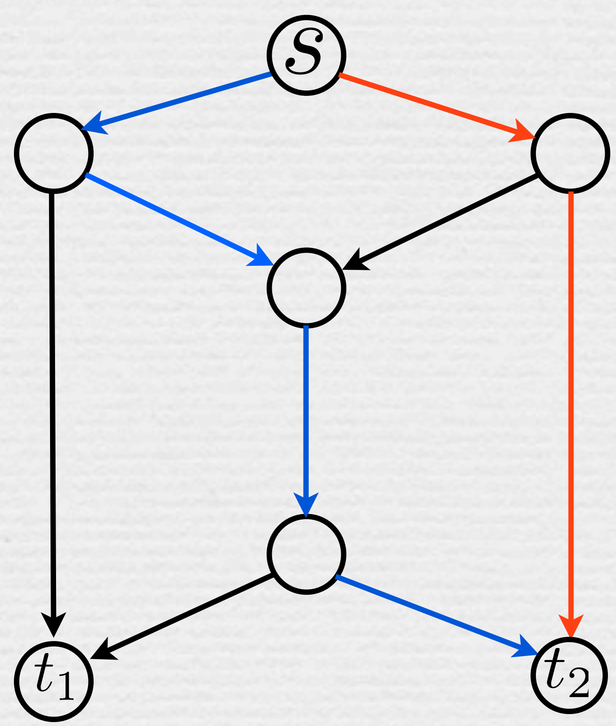
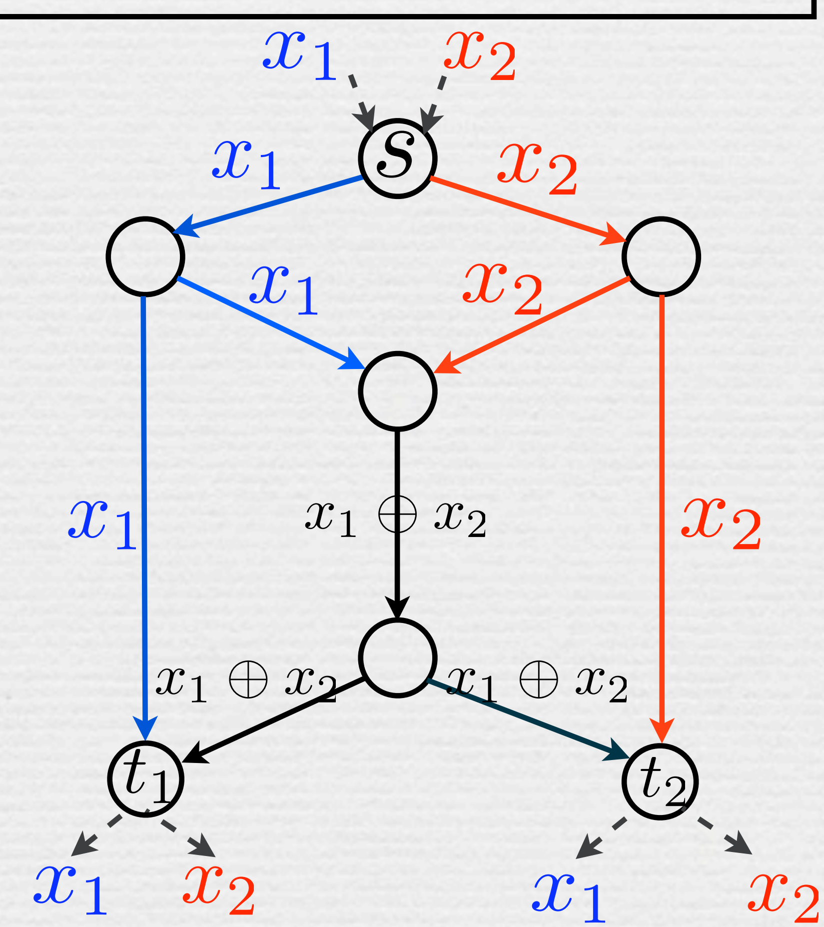
Example: condition ($\star$) holds for $r = 2$



2 edge-disjoint paths from s to t_1



2 edge-disjoint paths from s to t_2



solvable at rate 2 (coding is required)